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Extended Poincaré supersymmetry, rotation groups and branching rules

M Yang† and B G Wybourne

Department of Physics, University of Canterbury, Christchurch, New Zealand

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Abstract. The decomposition of the basic spin irreps of the special orthogonal group SO_{2k} into irreps of $SO_{D-2}\times K$, where D is the spacetime dimension of the extended D-dimensional Poincaré supersymmetry and K is the appropriate automorphism group, is determined. General results for $D\leqslant 10$ capable of extending decompositions of irreps giving rise to helicities greater than ± 2 are given together with a general method for $D>10$. Several new branching rules for subgroups of SO_{2k} are developed. A number of specific results are tabulated.

1. Introduction

The even-dimensional rotation groups SO_{2k} continue to find wide applications in physical problems. Important examples arise in the interacting boson model of nuclei (Arima and Iachello 1976), extended Poincaré supersymmetry (Strathdee 1985) and in superstring theories (Green and Schwarz 1984). The properties of the basic spin irreps of SO_n are of special significance. The analysis of the antisymmetric powers of a spin irrep is an important problem in supergravity theories (Curtright 1982a, b, Bergshoeff and de Roo 1982, 1984). A complete resolution of the second and third powers of the basic spin irrep of SO_n together with a prescription for analysing the fourth power of these irreps has been given (King *et al* 1981). A complete reduction of all antisymmetric powers of the basic spin irrep of SO_{10} has been derived (Black and Wybourne 1983).

The study of the properties of the even-dimensional rotation groups SO_{2k} is complicated by the occurrence of pairs of conjugate irreps that must be carefully distinguished at all stages. A compact description of the resolution of the Kronecker products of irreps of SO_{2k} has been given (Black *et al* 1983). Branching rules for irreps of SO_{2k} have in many cases been developed (King 1975, Black *et al* 1983, Black and Wybourne 1983). A method for $E_8\downarrow SO_{16}$ branching rules has also been given (Wybourne 1984).

In this paper we first give a complete resolution of all antisymmetric powers of the basic spin irreps of SO_n for $n\leqslant 10$. It then becomes possible to express any symmetrised power of the basic spin irreps for $n\leqslant 10$ as sums of products of the antisymmetrised powers. The symmetrised powers of the basic spin irreps, up to fourth power, are tabulated for $SO_n(n\leqslant 10)$.

We next introduce a number of additional branching rules for important subgroups of SO_{2k} drawing heavily upon the properties of Schur functions (King *et al* 1981, Black *et al* 1983).

† On leave from: Department of Applied Physics, Beijing, Polytechnic University, Beijing, China.

Finally we examine the decomposition of the basic spin irreps of SO_{2k} under the group-subgroup restriction $SO_{2k} \downarrow SO_{D-2} \times K$ where D is the spacetime dimension of the extended D-dimensional Poincaré supersymmetry and K is the automorphism group appropriate to D. This problem is of special significance in determining acceptable light-like representations. Group isomorphisms and automorphisms are exploited to give for $D \leqslant 10$ results that can readily be extended to any number of supercharges. A general method for cases with $D > 10$ is sketched. A useful table of decompositions is included.

2. Symmetrised powers of basic spin irreps of SO_n

Let us designate the 2^k-dimensional basic spin irrep of SO_{2k+1} by Δ and two inequivalent 2^{k-1}-dimensional basic spin irreps of SO_{2k} by Δ_+ and Δ_- (King *et al* 1981). The basic spin irreps Δ of SO_{2k+1} are orthogonal if $2k+1 = 1,7 (\bmod 8)$ or symplectic if $2k+1 = 3,5 (\bmod 8)$ while the basic spin irreps $\Delta_\pm$ of SO_{2k} are orthogonal if $2k = 0 (\bmod 8)$, symplectic if $2k = 4 (\bmod 8)$ and complex if $2k = 2,6 (\bmod 8)$.

In general the resolution of a symmetrised power of a basic spin irrep amounts to the evaluation of the spin plethysm (Littlewood 1947, 1948, 1950)

$$\Delta \otimes \{\lambda\} \qquad n = 2k+1 \tag{1a}$$

$$\Delta_\pm \otimes \{\lambda\} \qquad n = 2k \tag{1b}$$

where for the pth power in Δ (or $\Delta_\pm$) (λ) is a partition of the integer p, i.e. $\lambda \vdash p$.

The evaluation of the spin plethysms in (1) is equivalent to evaluating the branching rule for the unitary group irrep $\{\lambda\}$ under the restriction

$$U_{2^k} \downarrow SO_{2k+1} \qquad \text{where } \{1\} \downarrow \Delta \tag{2a}$$

or

$$U_{2^{k-1}} \downarrow SO_{2k} \qquad \text{where } \{1\} \downarrow \Delta_+ \text{ (or } \Delta_-) \tag{2b}$$

where in $(2b)$ if $\{1\} \downarrow \Delta_+$ then $\{\bar{1}\} \downarrow \Delta_-$. (In general we understand the irrep $\{\bar{\mu}\}$ of the unitary group to be contragradient to the irrep $\{\mu\}$.) The plethysms $\Delta \otimes \{1^x\}$ or $\Delta_\pm \otimes \{1^x\}$ correspond to the reduction of an antisymmetric tensor $\{1^x\}$ under the appropriate rotation group.

Let us put $\alpha = 2^k$ or 2^{k-1} according to whether we are concerned with SO_{2k+1} or SO_{2k} respectively. Suppose we take a set of α fermion fields spanning the basic spin irrep of $SO_n (n = 2k+1$ or $2k)$ and form all possible antisymmetric states. The total number of such states will be 2^α and the complete set of antisymmetric states will span the vector irrep $\{1\}$ of U_{2^α}.

Under $U_{2^\alpha} \downarrow SO_{2\alpha+1}$ we find (Wybourne 1973, 1974)

$$\{1\} \downarrow \Delta. \tag{3}$$

Further restriction to $SO_{2\alpha}$ gives

$$\Delta \downarrow \Delta_+ + \Delta_-. \tag{4}$$

Restriction to the subgroup $SU_\alpha \times U_1$ then leads to (Black and Wybourne 1983)

$$\Delta_\pm \downarrow \sum_{s_\pm} \{1^{\alpha - s_\pm}\} \times \{\tfrac{1}{2}\alpha - s_\pm\} \qquad \alpha \geqslant s_\pm \tag{5}$$

where the + or − sign is taken right through as appropriate and s_+ (s_-) are even (odd) integers.

The group SU_α may be further restricted by considering whether the basic spin irrep Δ or $\Delta_\pm$ of SO_n is orthogonal, symplectic or complex leading to the group structures listed in table 1 for $n \geq 5$.

Table 1. Group-subgroup structures for the set of antisymmetrised powers of a basic spin irrep of $SO_n (n \geq 5)$.

$n = 0,1,7 \pmod 8$	$U_{2^\alpha} \supset SO_{2\alpha+1} \supset SO_{2\alpha} \supset [SU_\alpha \supset SO_\alpha \supset SO_n] \times U_1$
$n = 3,4,5 \pmod 8$	$U_{2^\alpha} \supset SO_{2\alpha+1} \supset SO_{2\alpha} \supset [SU_\alpha \supset Sp_\alpha \supset SO_n] \times U_1$
$n = 2,6 \pmod 8$	$U_{2^\alpha} \supset SO_{2\alpha+1} \supset SO_{2\alpha} \supset [SU_\alpha \supset SO_n] \times U_1$

For $n \leq 6$ it is useful to exploit the properties of the locally isomorphic sets of groups

$$SO_2 \sim U_1 \tag{6a}$$

$$SU_2 \sim SO_3 \sim Sp_2 \tag{6b}$$

$$SO_4 \sim SU_2 \times SU_2 \sim SO_3 \times SO_3 \sim Sp_2 \times Sp_2 \tag{6c}$$

$$Sp_4 \sim SO_5 \tag{6d}$$

$$SO_6 \sim SU_4 \tag{6e}$$

and the representation correspondences

$$[a] \sim \{a\} \tag{7a}$$

$$\{a\} \sim [a/2] \sim \langle a \rangle \tag{7b}$$

$$[a, b] \sim \{a+b\} \times \{a-b\} \sim [(a+b)/2] \times [(a-b)/2] \sim \langle a+b \rangle \times \langle a-b \rangle \tag{7c}$$

$$\langle a, b \rangle \sim [(a+b)/2, (a-b)/2] \tag{7d}$$

$$[abc] \sim \{a+b, a-c, b-c\}. \tag{7e}$$

The three-index outer automorphism of SO_8 is such that (Littlewood 1948)

$$[abcd] \sim [\tfrac{1}{2}(a+b+c+d), \tfrac{1}{2}(a+b-c-d), \tfrac{1}{2}(a-b+c-d), \tfrac{1}{2}(-a+b+c-d)] \tag{8}$$

leading for example to

$$\Delta_+ \sim [1] \sim \Delta_- \sim \Delta_+. \tag{9}$$

Thus the automorphism applied to Δ_+ yields the vector irrep of SO_8. Two further applications of $\sim$ yields the original irrep. We shall designate two successive automorphisms by the symbol $\sim''$ noting that

$$[a, b, c, d] \sim'' [\tfrac{1}{2}(a+b+c-d), \tfrac{1}{2}(a+b-c+d), \tfrac{1}{2}(a-b+c+d), \tfrac{1}{2}(a-b-c-d)]. \tag{8'}$$

The above results together with those given by King *et al* (1981) and the Kronecker product results of Black *et al* (1983) readily lead to the resolution of the antisymmetrised powers of the basic spin irreps of SO_n for $n \leq 10$. A method of evaluating the corresponding powers for SO_{11} has been given (Black and Wybourne 1983) but the results are too voluminous to present here. In the case of $n = 2k$ it is only necessary to list the results up to $\Delta_+ \otimes \{1^k\}$ due to the involutary outer automorphism † for SO_{2k} which gives

$$\Delta_+ \otimes \{1^{2k-x}\} = (\Delta_+ \otimes \{1^x\})^\dagger \tag{10}$$

recalling that under †

$$([\lambda]_\pm)^\dagger = [\lambda]_\mp. \tag{11}$$

For SO_{2k+1} we have the equivalence

$$\Delta \otimes \{1^{2k+1-x}\} = \Delta \otimes \{1^x\}. \tag{12}$$

The results for $n \leqslant 10$ are collected together in table 2. The symmetrised powers, up to power four, of the basic spin irreps of SO_n with $n \leqslant 10$ are given in table 3.

Table 2. Antisymmetrised nth powers of the basic spin irreps of $SO_n (n \leqslant 10)$.

N	SO_2	SO_4	SO_6	SO_8	SO_{10}
1	Δ_+	Δ_+	Δ_+	Δ_+	Δ_+
2		$[0]$	$[1]$	$[1^2]$	$[1^3]$
3			Δ_-	$[\Delta, 1]_-$	$[\Delta; 1^2]_-$
4			$[0]$	$[2]+[1^4]_-$	$[2^2]+[21^4]_-$
5				$[\Delta; 1]_+$	$[\Delta; 21]_- + [\Delta; 1^5]_-$
6				$[1^2]$	$[2^21^3]_- + [31^2]$
7				Δ_-	$[\Delta; 21^2]_- + [\Delta; 3]_+$
8				$[0]$	$[4]+[2^3]+[31^3]$
N	SO_3	SO_5	SO_7		SO_9
1	Δ	Δ	Δ		Δ
2	$[0]$	$[1]+[0]$	$[1^2]+[1]$		$[1^3]+[1^2]$
3		Δ	$[\Delta; 1]+\Delta$		$[\Delta; 1^2]+[\Delta; 1]$
4		$[0]$	$[2]+[1^3]+[1]+[0]$		$[21^3]+[1^4]+[2^2]+[21]+[2]$
5			$[\Delta; 1]+\Delta$		$[\Delta; 21]+[\Delta; 2]+[\Delta; 1^4]+[\Delta; 1^2]+(\Delta; 1]$
6			$[1^2]+[1]$		$[2^21^2]+[21^3]+[31^2]+[31]+[21^2]+[21]$ $+[1^3]+[1^2]$
7			Δ		$[\Delta; 3]+[\Delta; 21^2]+[\Delta; 21]+[\Delta; 2]+[\Delta; 1^3]$ $+[\Delta; 1^2]+[\Delta; 1]+\Delta$
8			$[0]$		$[4]+[31^3]+[31^2]+[3]+[2^3]+[2^21]+[2^2]$ $+[21^3]+[21^2]+[2]+[1^4]+[1^3]+[1]+[0]$

3. Branching rules for subgroups of SO_n

Morris (1958, 1961) has given results for the decomposition of the basic spin irreps for the group-subgroup combinations

$$SO_{4rs} \supset SO_{2r} \times SO_{2s}$$

$$SO_{4rs+2s} \supset SO_{2r+1} \times SO_{2s}$$

$$SO_{4rs+2r+2s+1} \supset SO_{2r+1} \times SO_{2s+1}$$

while King (1975) gave equivalent results for the full orthogonal groups and for $O_{4rs} \supset Sp_{2r} \times Sp_{2s}$. Black and Wybourne (1983) have also given results for a number

Table 3. Symmetrised powers of basic spin irreps of SO_n.

$n=2k$	SO_4	SO_6	SO_8	SO_{10}
$\Delta_+\otimes\{2\}$	$[1^2]_+$	$[1^3]_+$	$[1^4]_+ + [0]$	$[1^5]_+ + [1]$
$\Delta_+\otimes\{1^2\}$	$[0]$	$[1]$	$[1^2]$	$[1^3]$
$\Delta_+\otimes\{3\}$	$[\Delta;1]_+$	$[\Delta;1^3]_+$	$[\Delta;1^4]_+ + \Delta_+$	$[\Delta;1^5]_+ + [\Delta;1]_+$
$\Delta_+\otimes\{21\}$	Δ_+	$[\Delta;1]_+$	$[\Delta;1^2]_+ + \Delta_+$	$[\Delta;1^3]_+ + [\Delta;1]_+ + \Delta_-$
$\Delta_+\otimes\{1^3\}$	—	Δ_-	$[\Delta;1]_-$	$[\Delta;1^2]_-$
$\Delta_+\otimes\{4\}$	$[2^2]_+$	$[2^3]_+$	$[2^4]_+ + [1^4]_+ + [0]$	$[2^5]_+ + [21^4]_+ + [2]$
$\Delta_+\otimes\{31\}$	$[1^2]_+$	$[21^2]_+$	$[2^21^2]_+ + [1^4]_+ + [1^2]$	$[2^31^2]_+ + [21^4]_+ + [21^2] + [1^4]_+$
$\Delta_+\otimes\{2^2\}$	$[0]$	$[2]$	$[2^2] + [1^4]_+ + [0]$	$[2^3] + [21^4]_+ + [2] + [1^4] + [0]$
$\Delta_+\otimes\{21^2\}$	—	$[1^2]$	$[21^2] + [1^2]$	$[2^21^2] + [21^2] + [1^4] + [1^2]$
$\Delta_+\otimes\{1^4\}$	—	$[0]$	$[2] + [1^4]_-$	$[2^2] + [21^4]_-$
$n=2k+1$	SO_3	SO_5	SO_7	SO_9
$\Delta\otimes\{2\}$	$[1]$	$[1^2]$	$[1^3] + [0]$	$[1^4] + [1] + [0]$
$\Delta\otimes\{1^2\}$	$[0]$	$[1] + [0]$	$[1^2] + [1]$	$[1^2] + [1]$
$\Delta\otimes\{3\}$	$[\Delta;1]$	$[\Delta;1^2]$	$[\Delta;1^3] + \Delta$	$[\Delta;1^4] + [\Delta;1] + \Delta$
$\Delta\otimes\{21\}$	Δ	$[\Delta;1] + \Delta$	$[\Delta;1^2] + [\Delta;1] + \Delta$	$[\Delta;1^3] + [\Delta;1^2] + [\Delta;1] + 2\Delta$
$\Delta\otimes\{1^3\}$	—	Δ	$[\Delta;1] + \Delta$	$[\Delta;1^2] + [\Delta;1]$
$\Delta\otimes\{4\}$	$[2]$	$[2^2]$	$[2^3] + [1^3] + [0]$	$[2^4] + [21^3] + [2] + [1^4] + [1] + [0]$
$\Delta\otimes\{31\}$	$[1]$	$[21] + [1^2]$	$[2^21] + [21^2] + [1^3]$ $+ [1^2] + [1]$	$[2^31] + [2^21^2] + [21^3] + [21^2] + [21]$ $+ 2[1^4] + 2[1^3] + 2[1^2] + [1]$
$\Delta\otimes\{2^2\}$	$[0]$	$[2] + [1] + [0]$	$[2^2] + [21] + [2]$ $+ [1^3] + [0]$	$[2^3] + [2^21] + [2^2] + [21^3] + [2]$ $+ 2[1^4] + [1^3] + [1] + 2[0]$
$\Delta\otimes\{21^2\}$	—	$[1^2] + [1]$	$[21^2] + [21] + [1^3]$ $+ 2[1^2] + [1]$	$[2^21^2] + [2^21] + [21^3] + 2[21^2]$ $+ [21] + [1^4] + 2[1^3] + 2[1^2] + [1]$
$\Delta\otimes\{1^4\}$	—	$[0]$	$[2] + [1^3] + [1] + [0]$	$[2^2] + [21^3] + [21] + [2] + [1^4]$

of subgroups for SO_{2k}. King's results may be extended to the special orthogonal group SO_n. For $n=2k$ it is necessary to make use of the theory of difference characters (Murnaghan 1983, Littlewood 1950, Butler and Wybourne 1969). In deriving our results we make extensive use of the properties of the infinite S-function series $A, B, C, \ldots$, defined elsewhere (Black *et al* 1983). We use ω_ζ to denote the sum of parts of a partition (ζ) and use $\zeta_+(\zeta_-)$ to stand for any S function with ω_ζ even (odd).

The relevant results are collected together in table 4. The format of these tables follows that of Black and Wybourne (1983). The results for the decomposition of the basic spin irreps are of crucial importance in establishing our subsequent results. Note that in each case the embedding is specified by the assumed decomposition of the vector irrep [1].

4. Branching rules for the extended D-dimensional Poincaré supersymmetry

The connection between Clifford and Lie algebras is well known. The Clifford algebra C_k contains the important Lie subalgebras

$$C_k \supset SO_{2k+1} \supset SO_{2k}.$$

The 2^k-dimensional vector space on which C_k acts is irreducible under SO_{2k+1} and separates into two inequivalent 2^{k-1}-dimensional irreps under SO_{2k}. The restriction

Table 4. Branching rules for SO_n.

$\mathrm{SO}(4rs)\downarrow \mathrm{Sp}(2r)\times\mathrm{Sp}(2s)$	
$[1]$	$\langle 1\rangle\times\langle 1\rangle$
Δ	$\sum_{\zeta}\langle s^r/\zeta\rangle\times\langle\tilde{\zeta}\rangle$
Δ''	$\sum_{\zeta}(-1)^{\omega_\zeta}\langle s^r/\zeta\rangle\times\langle\tilde{\zeta}\rangle$
$\Delta_\pm$	$\sum_{\zeta\pm}\langle s^r/\zeta_\pm\rangle\times\langle\tilde{\zeta}_\pm\rangle$ ($\zeta_\pm$ is partition of even (odd) weight)
$[\lambda]$	$\sum_{\eta}\langle((\lambda/C)\circ\eta)/B\rangle\times\langle\eta/B\rangle$
$\Box$	$\sum_{\eta\vdash 2rs}\langle\eta/B\rangle\times\langle\tilde{\eta}/B\rangle$
$\Box''$	$\sum_{\zeta,\eta,\sigma,\tau}(-1)^{\omega_\zeta}\langle(s^r/\zeta\sigma)\cdot(s^r/\eta\sigma)\rangle\langle(\tilde{\zeta}/\tau)\cdot(\tilde{\eta}/\tau)\rangle$
$\Box_\pm$	$\sum_{\zeta_\pm,\eta_\pm,\sigma,\tau}\langle(s^r/\zeta_\pm\sigma)\cdot(s^r/\eta_\pm\sigma)\rangle\times\langle(\tilde{\zeta}_\pm/\tau)\cdot(\tilde{\eta}_\pm/\tau)\rangle$
	$-\sum_{\eta\vdash 2rs-2-m,m}\langle\eta/B\rangle\times\langle\tilde{\eta}/B\rangle$ (m is positive integer)
$[\Delta;\lambda]$	$\sum_{\zeta,\eta,\sigma,\tau}\langle(s^r/\zeta\sigma)\cdot(((\lambda/E)\circ\eta)/B\sigma)\rangle\times\langle(\tilde{\zeta}/\tau)\cdot(\eta/B\tau)\rangle$
$[\Delta;\lambda]''$	$\sum_{\zeta,\eta,\sigma,\tau}(-1)^{\omega_\zeta}\langle(s^r/\zeta\sigma)\cdot(((\lambda/G)\circ\eta)/B\sigma)\rangle\times\langle\tilde{\zeta}/\tau)\circ(\eta/B\tau)\rangle$
$[\Delta;\lambda]_\pm$	$\sum_{m,\zeta_{\pm(-)^m},\eta,\sigma,\tau}(-1)^m\langle(s^r/\zeta_{\pm(-)^m}\sigma)\cdot(((\lambda/Cm)\circ\eta)/\sigma B)\rangle\times\langle(\zeta_{\pm(-)^m}/\tau)\cdot(\eta/B\tau)\rangle$
$[\Box,\lambda]''$	$\sum_{\zeta,\eta,\rho,\sigma,\tau,\nu,\theta}(-1)^{\omega_\eta}\langle(((s^r/\zeta\sigma)\cdot(s^r/\eta\sigma))/\nu)\cdot(((\lambda/A)\circ\rho)/B\nu)\rangle\times\langle(((\zeta/\tau)\cdot(\tilde{\eta}/\tau))/\theta)\cdot(\rho/B\theta)\rangle$
$\mathrm{SO}(4rs+2r+2s+1)\downarrow\mathrm{SO}(2r+1)\times\mathrm{SO}(2s+1)$	
$[1]$	$[1]\times[1]$
Δ	$\sum_{\zeta}[\Delta;s^r/\zeta]\times[\Delta;\tilde{\zeta}]$
$[\lambda]$	$\sum_{\eta}[((\lambda/C)\circ\eta)/D]\times[\eta/D]$
$[\Delta;\lambda]$	$\sum_{\zeta\eta,\sigma\tau}[\Delta;(s^r/\zeta\sigma)\cdot(((\lambda/E)\circ\eta)/F\sigma)]\times[\Delta;(\tilde{\zeta}/\tau)\cdot(\eta/F\tau)]$
$\mathrm{SO}(4rs+2s)\downarrow\mathrm{SO}(2r+1)\times\mathrm{SO}(2s)$	
$[1]$	$[1]\times[1]$
Δ	$\sum_{\zeta}[s^r/\zeta]\times[\Delta;\tilde{\zeta}]$
Δ''	$\sum_{\zeta}(-1)^{\omega_\zeta}[s^r/\zeta]\times[\Delta;\tilde{\zeta}]''$
$\Delta_\pm$	$\sum_{\zeta}[s^r/\zeta]\times[\Delta;\tilde{\zeta}]_{\pm(-)^{\omega_\zeta}}$
$[\lambda]$	$\sum_{\eta}[((\lambda/C)\circ\eta)/D]\times[\eta/D]$
$\Box$	$\sum_{\eta\vdash 2rs+s}[\eta/D]\times[\tilde{\eta}/D]$
$\Box''$	$\sum_{m,\zeta,\eta,\sigma,\tau}(-1)^{\omega_\zeta}[(s^r/\eta\sigma)\cdot(s^r/\zeta\sigma)]\times[\Box;(\tilde{\eta}/\tau Q)\cdot(\tilde{\zeta}/\tau L)]''$
$\Box_\pm$	$\sum_{\zeta,\eta,\sigma,\rho}[(s^r/\zeta\sigma)\cdot(s^r/\eta\sigma)]\times[\bar{\tau};(\tilde{\zeta}/\rho\tau B)\cdot(\tilde{\eta}/\rho)\cdot Q_{\pm(-)^{\omega_\zeta}}+s]_{(-)^{\omega_\eta}}$
	$-\sum_{m,\eta\vdash 2rs+s-2-2m}[\eta/D][\tilde{\eta}/D]$
$[\Delta,\lambda]$	$\sum_{\zeta,\eta,\sigma,\tau,\theta}[(s^r/\zeta\sigma)\cdot(((\lambda/E)\circ\eta)/D\sigma)]\times[\Delta;(\tilde{\zeta}/\theta)\cdot(\eta/F\theta)]$

Table 4. (continued)

$\mathrm{SO}(4rs+2s)\downarrow\mathrm{SO}(2r+1)\times\mathrm{SO}(2s)$	
$[\Delta,\lambda]''$	$\sum_{\zeta,\eta,\sigma,\tau,\theta}(-1)^{\omega_\zeta}[(s^r/\zeta\sigma)\cdot(((\lambda/G)\circ\eta)/D\sigma)]\times[\Delta;(\tilde{\zeta}/\theta)\cdot(\eta/H\theta)]$
$[\Delta,\lambda]_\pm$	$\sum_{m,n,\zeta,\eta,\sigma,\tau,\theta}(-1)^m[(s^r/\zeta\sigma)\cdot(((\lambda/Cm)\circ\eta)/D)]\times[\Delta;(\tilde{\zeta}/\theta)\cdot(\eta/D1^n\theta)]_{\pm(-)^{\omega_\zeta+m+n}}$
$[\Delta,\lambda]''$	$\sum_{\zeta,\eta,\sigma,\tau,\theta,\rho}(-1)^{\omega_\zeta}[(((s^r/\eta\sigma)\cdot(s^r/\zeta\sigma))/\theta)\cdot(((\lambda/A)\circ\eta)/D\theta)]$ $\times[\square;(((\tilde{\eta}/\tau Q)\cdot(\tilde{\zeta}/\tau L))/\rho)\cdot(\eta/B\rho)]''$
$\mathrm{SO}(4rs)\downarrow\mathrm{SO}(2r)\times\mathrm{SO}(2s)$	
$[1]$	$[1]\times[1]$
Δ	$\sum_\zeta[s^r/\zeta]\times[\tilde{\zeta}]$
Δ''	$\sum_\zeta(-1)^{\omega_\zeta}[s^r/\zeta]\times[\tilde{\zeta}]$
$\Delta_\pm$	$\sum_{\zeta\pm}[s^r/\zeta_\pm]\times[\tilde{\zeta}_\pm]$
$[\lambda]$	$\sum_\eta[((\lambda/C)\circ\eta)/D]\times[\eta/D]$
$\square$	$\sum_{\eta\vdash 2rs}[\eta/D]\times[\tilde{\eta}/D]$
$\square''$	$\sum_{\zeta,\eta,\sigma,\tau}(-1)^{\omega_\eta}[(s^r/\zeta\sigma)\cdot(s^r/\eta\sigma)]\times[(\tilde{\zeta}/\tau)\cdot(\tilde{\eta}/\tau)]$
$\square_\pm$	$\sum_{\zeta_\pm,\eta_\pm,\sigma,\tau}[(s^r/\zeta_\pm\sigma)\cdot(s^r/\eta_\pm\sigma)]\times[(\tilde{\zeta}_\pm/\tau)\cdot(\tilde{\eta}_\pm/\tau)]-\sum_{\eta\vdash 2rs-2-m,m}[\eta/D]\times[\tilde{\eta}/D]$ (m is positive integer)
$[\Delta;\lambda]$	$\sum_{\zeta,\eta,\sigma,\tau}[(s^r/\zeta\sigma)\cdot(((\lambda/E)\circ\eta)/D\sigma)]\times[\tilde{\zeta}/\tau)\cdot(\eta/D\tau)]$
$[\Delta;\lambda]''$	$\sum_{\zeta,\eta,\sigma,\tau}(-1)^{\omega_\zeta}[(s^r/\zeta\sigma)\cdot(((\lambda/G)\circ\eta)/D\sigma)]\times[(\tilde{\zeta}/\tau)\cdot(\eta/D\tau)]$
$[\Delta;\lambda]_\pm$	$\sum_{m,\zeta_{\pm(-)^m},\eta,\sigma,\tau}(-1)^m[(s^r/\zeta_{\pm(-)^m}\sigma)\cdot(((\lambda/Cm)\circ\eta)/\sigma D)]\times[(\zeta_{\pm(-)^m}/\tau)\cdot(\eta/D\tau)]$
$[\square;\lambda]$	$\sum_{\zeta,\eta,\rho,\sigma,\tau,\nu,\theta}(-1)^{\omega_\eta}[(((s^r/\zeta\sigma)\cdot(s^r/\eta\sigma))/\nu)\cdot(((\lambda/A)\circ\rho)/D\nu)]$ $\times[(((\tilde{\zeta}/\tau)\cdot(\tilde{\eta}/\tau))/\theta)\cdot(\rho/D\theta)]$

$SO_{2k+1}\downarrow SO_{2k}$ again leads to

$$\Delta\downarrow\Delta_+ + \Delta_-. \tag{13}$$

The irreps $\Delta_\pm$ of SO_{2k} may be further reduced by the action of subgroups of SO_{2k}. Strathdee (1985) has shown that for the extended D-dimensional Poincaré supersymmetry the group–subgroup structure

$$SO_{2k}\supset SO_{D-2}\times K \tag{14}$$

is of special significance in determining acceptable light-like representations. The choice of K is dictated by D. The group K is the automorphism group (or bosonic little group) which treats the supercharges $Q_j\,(j=1,\ldots,N)$ as an N vector.

We let $Q_{1/2}$ represent a set of real supercharges with $2k$ real components. This set generates the 2^k-dimensional Clifford algebra. The $Q_{1/2}$ span the vector irrep [1] of SO_{2k}. Under the restriction $SO_{2k}\downarrow SO_{D-2}\times K$ they span the basic spin irreps of SO_{D-2}

and the vector irreps of K. The relevant decompositions for the vector irrep [1] of SO_{2k} are listed in table 5 along with the relevant group-subgroup structures.

The decomposition of the vector irrep [1] of SO_{2k} fixes the group-subgroup embedding and leaves the corresponding decompositions for the basic spin irreps $\Delta_\pm$ of SO_{2k} to be determined.

Table 5. Decomposition of the vector irrep [1] of $SO_{2k} \downarrow SO_{D-2} \times K$.

$D=0,4 \pmod 8$	$SO_{2k} \downarrow SO_{D-2} \times SU_N \times U_1$	$2k=2^{(D-2)/2}N$
	$[1] \quad \downarrow \Delta_+ \times \{1\} \times \{1\} + \Delta_- \times \{\bar{1}\} \times \{\bar{1}\}$	
$D=1,3 \pmod 8$	$SO_{2k} \downarrow SO_{D-2} \times SO_N$	$2k=2^{(D-3)/2}N$
	$[1] \quad \downarrow \Delta \times [1]$	
$D=5,7 \pmod 8$	$SO_{2k} \downarrow SO_{D-2} \times Sp_N$	$2k=2^{(D-3)/2}N$
	$[1] \quad \downarrow \Delta \times \langle 1 \rangle$	
$D=2 \pmod 8$	$SO_{2k} \downarrow SO_{D-2} \times SO_{N_+} \times SO_{N_-}$	$2k=2^{(D-4)/2}(N_+ + N_-)$
	$[1] \quad \downarrow \Delta_+ \times [1] \times [0] + \Delta_- \times [0] \times [1]$	
	$SO_{2k} \downarrow SO_{D-2} \times SO_N$	
	$[1] \quad \downarrow \Delta_+ \times [1]$	
$D=6 \pmod 8$	$SO_{2k} \downarrow SO_{D-2} \times Sp_{N_+} \times Sp_{N_-}$	$2k=2^{(D-4)/2}(N_+ + N_-)$
	$[1] \quad \downarrow \Delta_+ \times \langle 1 \rangle \times \langle 0 \rangle + \Delta_- \times \langle 0 \rangle \times \langle 1 \rangle$	
	$SO_{2k} \downarrow SO_{D-2} \times Sp_N$	
	$[1] \quad \downarrow \Delta_+ \times \langle 1 \rangle$	

For $D=0,4 \pmod 8$ the required general result may be readily found by noting the equivalent group decompositions

$$\begin{tikzcd} SO_{2k} \arrow[rr] \arrow[dr] & & SO_{D-2} \times SU_N \times U_1 \\ & SU_k \times U_1 \arrow[ur] & \end{tikzcd} \tag{15}$$

leading to

$$\begin{aligned} & D=0,4 \pmod 8 \qquad 2k=2^{(D-2)/2}N \\ & SO_{2k} \downarrow SO_{D-2} \times SU_N \times U_1 \\ & \Delta_\pm \downarrow \sum_{s_\pm} \sum_{\rho \vdash s_\pm} [\Delta_+ \otimes \{\rho\}] \times \{\tilde{\rho}\} \times \{s_\pm - k/2\}. \end{aligned} \tag{16}$$

Use may be made of (10) to restrict the values of $s_\pm \leqslant k$. Equation (16) simplifies for $D=4$ and 8. For $D=4$ we have

$$\begin{aligned} & SO_{2N} \downarrow SO_2 \times SU_N \times U_1 \\ & \Delta_\pm \downarrow \sum_{s_\pm} [s_\pm]_+ \times \{1^{s_\pm}\} \times \{s_\pm - N/2\} \end{aligned} \tag{17}$$

while for $D=8$ exploitation of the local isomorphism $SO_6 \sim SU_4$ leads to

$$\begin{aligned} & SO_{8N} \downarrow SO_6 \times SU_N \times U_1 \\ & \Delta_\pm \downarrow \sum_{s_\pm}^{4N} \sum_{\rho \vdash s_\pm}^{\bullet} [\{\rho\}] \times \{\tilde{\rho}\} \times \{s_\pm - 2N\} \end{aligned} \tag{18}$$

where no part of (ρ) exceeds N nor does the number of parts of (ρ) exceed 4 and

$$\{\rho_1\rho_2\rho_3\rho_4\} \equiv \{\rho_1-\rho_4, \rho_2-\rho_4, \rho_3-\rho_4, 0\} \qquad \rho_4 \neq 0. \tag{19}$$

The above restrictions limit $\{\rho\}$ to standard inequivalent irreps of SU_4 involving not more than three parts which may be transcribed into the appropriate SO_6 irrep labels using $(7e)$.

Using the branching rules of table 4 together with the isomorphisms and automorphisms discussed in § 2 it was possible to obtain general results for $D=4$ to 10 as given in table 6. Note that for $D=6$ the summation $\Sigma_{s^+_\pm, s^-_\pm}$ is to be understood as first being made over (s^+_+, s^-_+) and then over (s^+_-, s^-_-) while for $\Sigma_{s^+_\pm, s^-_\mp}$ the sum is first made over (s^+_+, s^-_-) and then over (s^+_-, s^-_+) where $s^+_\pm \leq n_+$ and $s^-_\pm \leq n_-$ and as usual s_+ refers to even integers and s_- to odd integers. For $D=7$ $[\langle\lambda\rangle]$ is first evaluated in Sp_4 and then transcribed as a SO_5 irrep label using $(7d)$. Finally in the case of $D=10$ we note the presence of the $\sim''$ symbol which must be applied to each SO_8 irrep that arises in the summations. The case of $D=9$ can be found from those of $D=10$ by making use of the fact that under $SO_8 \downarrow SO_7$, $\Delta_+ \downarrow \Delta$. Some detailed results are given in table 7.

Table 6. Branching rules for the basic spin irreps of $SO_{2k} \downarrow SO_{D-2} \times K$.

$D=4$	SO_{2N}	$\downarrow SO_2 \times SU_N \times U_1$		
	$\Delta_\pm$	$\downarrow \sum_{s_\pm}^{N} [s_\pm]_+ \times \{1^{s_\pm}\} \times \{s_\pm - N/2\}$		
$D=5$	SO_{4n}	$\downarrow SO_3 \times Sp_{2n}$		$N=2n$
	$\Delta_\pm$	$\downarrow \sum_{s_\pm}^{n} [(n-s_\pm)/2] \times \langle 1^{s_\pm}\rangle$		
$D=6$	SO_{4n}	$\downarrow SO_4 \times Sp_{2n}$		$N=2n$
	$\Delta_\pm$	$\downarrow \sum_{s_\pm}^{n} [(n-s_\pm)/2, (n-s_\pm)/2] \times \langle 1^{s_\pm}\rangle$		
	$SO_{4(n_++n_-)}$	$\downarrow SO_4 \times Sp_{2n_+} \times Sp_{2n_-}$		$N=2(n_++n_-)$
	Δ_+	$\downarrow \sum_{s^+_\pm, s^-_\pm}^{n_-n_+} [(n_++n_--s^+_\pm-s^-_\pm)/2, (n_+-n_--s^+_\pm+s^-_\pm)/2]$ $\times \langle 1^{s^+_\pm}\rangle \times \langle 1^{s^-_\pm}\rangle$		
	Δ_-	$\downarrow \sum_{s^+_\pm, s^-_\mp}^{n_+n_-} [(n_++n_--s^+_\pm-s^-_\mp)/2, (n_+-n_--s^+_\pm+s^-_\mp)/2]$ $\times \langle 1^{s^+_\pm}\rangle \times \langle 1^{s^-_\mp}\rangle$		
$D=7$	SO_{8n}	$\downarrow SO_5 \times Sp_{2n}$		$N=2n$
	$\Delta_\pm$	$\downarrow \sum_{\zeta_\pm} [\langle n^2/\zeta_\pm\rangle] \times \langle\tilde{\zeta}_\pm\rangle$		
$D=8$	SO_{8N}	$\downarrow SO_6 \times SU_N \times U_1$		
	$\Delta_\pm$	$\downarrow \sum_{s_\pm}^{4n} \sum_{\zeta \vdash s_\pm} [\{\tilde{\zeta}\}] \times \{\zeta\} \times \{s_\pm - 2N\}$		
$D=10$	SO_{16n+8}	$\downarrow SO_8 \times SO_{2n+1}$		$N=2n+1$
	$\Delta_\pm$	$\downarrow \sum_{\zeta} [\Delta; \tilde{\zeta}'']_{\pm(-)\omega_\zeta} \times [4^n/\tilde{\zeta}]$		
	SO_{16n}	$\downarrow SO_8 \times SO_{2n}$		$N=2n$
	$\Delta_\pm$	$\downarrow \sum_{\zeta_\pm} ([n^{\tilde{4}''}/\zeta_\pm]_+ + [n^{\tilde{4}''}/\zeta_\pm]_-) \times [\tilde{\zeta}_\pm]$	$\zeta_1 < n$	
		$+\sum_{\zeta_\pm} [n^{\tilde{4}''}/\zeta_\pm] \times ([\tilde{\zeta}_\pm]_+ + [\tilde{\zeta}_\pm]_-)$	$\zeta_1 = n$	

Table 7. Branching rules for basic spin irreps for $D = 5$ to 10.

$D=5$	$N=2$	SO_4	$\downarrow SO_3 \times Sp_2$
		Δ_+	$\downarrow \Delta\langle 0\rangle$
		Δ_-	$\downarrow [0]\langle 1\rangle$
	$N=4$	SO_8	$\downarrow SO_3 \times Sp_4$
		Δ_+	$\downarrow [1]\langle 0\rangle + [1]\langle 1^2\rangle$
		Δ_-	$\downarrow \Delta\langle 1\rangle$
	$N=6$	SO_{12}	$\downarrow SO_3 \times Sp_6$
		Δ_+	$\downarrow [\Delta;\, 1]\langle 0\rangle + \Delta\langle 1^2\rangle$
		Δ_-	$\downarrow [1]\langle 1\rangle + [0]\langle 1^3\rangle$
	$N=8$	SO_{16}	$\downarrow SO_3 \times Sp_8$
		Δ_+	$\downarrow [2]\langle 0\rangle + [1]\langle 1^2\rangle + [0]\langle 1^4\rangle$
		Δ_-	$\downarrow [\Delta;\, 1]\langle 1\rangle + \Delta\langle 1^3\rangle$
	$N=10$	SO_{20}	$\downarrow SO_3 \times Sp_{10}$
		Δ_+	$\downarrow [\Delta;\, 2]\langle 0\rangle + [\Delta;\, 1]\langle 1^2\rangle + \Delta\langle 1^4\rangle$
		Δ_-	$\downarrow [2]\langle 1\rangle + [1]\langle 1^3\rangle + [0]\langle 1^5\rangle$
	$N=12$	SO_{24}	$\downarrow SO_3 \times Sp_{12}$
		Δ_+	$\downarrow [3]\langle 0\rangle + [2]\langle 1^2\rangle + [1]\langle 1^4\rangle + [0]\langle 1^6\rangle$
		Δ_-	$\downarrow [\Delta;\, 2]\langle 1\rangle + [\Delta;\, 1]\langle 1^3\rangle + \Delta\langle 1^5\rangle$
$D=6$	$N=2$	SO_4	$\downarrow SO_4 \times Sp_2$
		Δ_+	$\downarrow \Delta_+\langle 0\rangle$
		Δ_-	$\downarrow [0]\langle 1\rangle$
	$N=4$	SO_8	$\downarrow SO_4 \times Sp_4$
		Δ_+	$\downarrow [1^2]_+\langle 0\rangle + [0]\langle 1^2\rangle$
		Δ_-	$\downarrow \Delta_+\langle 1\rangle$
		SO_8	$\downarrow SO_4 \times Sp_2 \times Sp_2$
		Δ_+	$\downarrow [1]\langle 0\rangle\langle 0\rangle + [0]\langle 1\rangle\langle 1\rangle$
		Δ_-	$\downarrow \Delta_+\langle 0\rangle\langle 1\rangle + \Delta_-\langle 1\rangle\langle 0\rangle$
	$N=6$	SO_{12}	$\downarrow SO_4 \times Sp_6$
		Δ_+	$\downarrow [\Delta;\, 1^2]_+\langle 0\rangle + \Delta_+\langle 1^2\rangle$
		Δ_-	$\downarrow [1^2]_+\langle 1\rangle + [0]\langle 1^3\rangle$
		SO_{12}	$\downarrow SO_4 \times Sp_4 \times Sp_2$
		Δ_+	$\downarrow [\Delta;\, 1]_+\langle 0\rangle\langle 0\rangle + \Delta_+\langle 1\rangle\langle 1\rangle + \Delta_-\langle 1^2\rangle\langle 0\rangle$
		Δ_-	$\downarrow [1^2]_+\langle 0\rangle\langle 1\rangle + [0]\langle 1^2\rangle\langle 1\rangle + [1]\langle 1\rangle\langle 0\rangle$
	$N=8$	SO_{16}	$\downarrow SO_4 \times Sp_8$
		Δ_+	$\downarrow [2^2]_+\langle 0\rangle + [1^2]_+\langle 1^2\rangle + [0]\langle 1^4\rangle$
		Δ_-	$\downarrow [\Delta;\, 1^2]_+\langle 1\rangle + \Delta_+\langle 1^3\rangle$
		SO_{16}	$\downarrow SO_4 \times Sp_6 \times Sp_2$
		Δ_+	$\downarrow [21]_+\langle 0\rangle\langle 0\rangle + [1]\langle 1^2\rangle\langle 0\rangle + [1^2]_+\langle 1\rangle\langle 1\rangle + [0]\langle 1^3\rangle\langle 1\rangle$
		Δ_-	$\downarrow [\Delta;\, 1^2]_+\langle 0\rangle\langle 1\rangle + \Delta_+\langle 1^2\rangle\langle 1\rangle + [\Delta;\, 1]_+\langle 1\rangle\langle 0\rangle + \Delta_-\langle 1^3\rangle\langle 0\rangle$
		SO_{16}	$\downarrow SO_4 \times Sp_4 \times Sp_4$
		Δ_+	$\downarrow [2]\langle 0\rangle\langle 0\rangle + [1^2]_-\langle 1^2\rangle\langle 0\rangle + [1^2]_+\langle 0\rangle\langle 1^2\rangle + [0]\langle 1^2\rangle\langle 1^2\rangle$
		Δ_-	$\downarrow [\Delta;\, 1]_+\langle 0\rangle\langle 1\rangle + [\Delta;\, 1]_-\langle 1\rangle\langle 0\rangle + \Delta_-\langle 1^2\rangle\langle 1\rangle + \Delta_+\langle 1\rangle\langle 1^2\rangle$
$D=7$	$N=2$	SO_8	$\downarrow SO_5 \times Sp_2$
		Δ_+	$\downarrow [1]\langle 0\rangle + [0]\langle 2\rangle$
		Δ_-	$\downarrow \Delta\langle 1\rangle$
	$N=4$	SO_{16}	$\downarrow SO_5 \times Sp_4$
		Δ_+	$\downarrow [2]\langle 0\rangle + [1^2]\langle 1^2\rangle + [1]\langle 2\rangle$
		Δ_-	$\downarrow [\Delta;\, 1]\langle 1\rangle + \Delta\langle 21\rangle$

Table 7. (continued)

	$N=6$	SO_{24}	$\downarrow SO_5\times Sp_6$
		Δ_+	$\downarrow [3]\langle 0\rangle+[21]\langle 1^2\rangle+[2]\langle 2\rangle+[1^2]\langle 21^2\rangle+[1]\langle 2^2\rangle+[0]\langle 2^3\rangle$
		Δ_-	$\downarrow [\Delta;\,2]\langle 1\rangle+[\Delta;\,1^2]\langle 1^3\rangle+[\Delta;\,1]\langle 21\rangle+\Delta\langle 2^21\rangle$
	$N=8$	SO_{32}	$\downarrow SO_5\times Sp_8$
		Δ_+	$\downarrow [4]\langle 0\rangle+[31]\langle 1^2\rangle+[3]\langle 2\rangle+[2^2]\langle 1^4\rangle+[21]\langle 21^2\rangle+[2]\langle 2^2\rangle+[1^2]\langle 2^21^2\rangle$
			$+[1]\langle 2^3\rangle+[0]\langle 2^4\rangle$
		Δ_-	$\downarrow [\Delta;\,3]\langle 1\rangle+[\Delta;\,21]\langle 3\rangle+[\Delta;\,2]\langle 21\rangle+[\Delta;\,1^2]\langle 21^3\rangle+[\Delta;\,1]\langle 2^21\rangle+\Delta\langle 2^31\rangle$
$D=8$	$N=1$	SO_8	$\downarrow SO_6\times U_1$
		Δ_+	$\downarrow [0]\{\bar{2}\}+[1]\{0\}+[0]\{2\}$
		Δ_-	$\downarrow \Delta_+\{\bar{1}\}+\Delta_-\{1\}$
	$N=2$	SO_{16}	$\downarrow SO_6\times SU_2\times U_1$
		Δ_+	$\downarrow [0]\{0\}(\{\bar{4}\}+\{4\})+[1^3]_+\{0\}\{\bar{2}\}+[1^3]_-\{0\}\{2\}+[1]\{2\}(\{\bar{2}\}+\{2\})$
			$+([0]\{4\}+[2]\{0\}+[1^2]\{2\})\{0\}$
		Δ_-	$\downarrow \Delta_+\{1\}\{\bar{3}\}+\Delta_-\{1\}\{3\}+\Delta_+\{3\}\{1\}+\Delta_-\{3\}\{\bar{1}\}+[\Delta;\,1]_+\{1\}\{\bar{1}\}$
			$+[\Delta;\,1]_-\{1\}\{1\}$
	$N=3$	SO_{24}	$\downarrow SO_6\times SU_3\times U_1$
		Δ_+	$\downarrow [0]\{0\}(\{\bar{6}\}+\{6\})+([1]\{2\}+[1^3]_+\{1^2\})\{\bar{4}\}+([1]\{2^2\}+[1^3]_-\{1\})\{4\}$
			$+([21^2]_+\{1\}+[2]\{2^2\}+[1^2]\{31\})\{\bar{2}\}+([21^2]_-\{1^2\}+[2]\{2\}+[1^2]\{32\})$
			$\{2\}+([3]\{0\}+[1^3]_+\{3\}+[1^3]_-\{3^2\}+[1]\{42\}+[21]\{21\})\{0\}$
		Δ_-	$\downarrow \Delta_-\{1\}\{\bar{5}\}+\Delta_+\{1^2\}\{5\}+([\Delta;\,1^3]_+\{0\}+\Delta_-\{3\}+[\Delta;\,1]_+\{21\})\{3\}$
			$+([\Delta;\,1^3]_-\{0\}+\Delta_+\{3^2\}+[\Delta;\,1]_-\{21\})\{\bar{3}\}$
			$+(\Delta_-\{41\}+[\Delta;\,1]_+\{1^2\}+[\Delta;\,1]_-\{32\}+[\Delta;\,1^2]_+\{21\})\{\bar{1}\}$
			$+(\Delta_+\{43\}+[\Delta;\,1]_-\{1\}+[\Delta;\,1]_+\{31\}+[\Delta;\,1^2]_-\{21\})\{1\}$
$D=9$	$N=1$	SO_8	$\downarrow SO_7\times SO_1$
		Δ_+	$\downarrow \Delta$
		Δ_-	$\downarrow [1]+[0]$
	$N=2$	SO_{16}	$\downarrow SO_7\times SO_2$
		Δ_+	$\downarrow ([1^3]+[2]+[1]+[0])[0]+([1^2]+[1])([2]_++[2]_-)+[0]([4]_++[4]_-)$
		Δ_-	$\downarrow ([\Delta;\,1]+\Delta)([1]_++[1]_-)+\Delta([3]_++[3]_-)$
	$N=3$	SO_{24}	$\downarrow SO_7\times SO_3$
		Δ_+	$\downarrow ([1]+[0])[4]+([\Delta;\,1]+\Delta)[3]+([\Delta;\,1^2]+[\Delta;\,1])[2]$
			$+([\Delta;\,2]+[\Delta;\,1]+\Delta)[1]+[\Delta;\,1^3][0]$
		Δ_-	$\downarrow ([1]+[0])[4]+([1^3]+[1^2])[3]+([21]+[2]+[1^2]+[1])[2]$
			$+([21^2]+[1^3])[1]+([3]+[2]+[1]+[0])[0]$
	$N=4$	SO_{32}	$\downarrow SO_7\times SO_4$
		Δ_+	$\downarrow ([2^3]+[4]+[3]+[2]+[1]+[0])[0]+([31^2]+[21^2]+[1^3])([1^2]_++[1^2]_-)$
			$+([2^21]+[21^2]+[31]+[3]+[21]+[2]+[1^2]+[1])[2]$
			$+([2^2]+[21]+[2])([2^2]_++[2^2]_-)+([21^2]+[21]+[1^3]+[1^2])$
			$([31]_++[31]_-)+([1^3]+[2]+[1]+[0])[4]+[1^3]$
			$([3^2]_++[3^2]_-)+([1^2]+[1])([42]_++[42]_-)+[0]([4^2]_++[4^2]_-)$
		Δ_-	$\downarrow ([\Delta;\,21^2]+[\Delta;\,1^3]+[\Delta;\,3]+[\Delta;\,2]+[\Delta;\,1]+\Delta)[1]$
			$+([\Delta;\,21]+[\Delta;\,2]+[\Delta;\,1^2]+[\Delta;\,1])([21]_++[21]_-)$
			$+([\Delta;\,1^3]+[\Delta;\,1^2]+[\Delta;\,2]+[\Delta;\,1]+\Delta)[3]+([\Delta;\,1^2]+[\Delta;\,1])$
			$([32]_++[32]_-)+([\Delta;\,1]+\Delta)([41]_++[41]_-)+\Delta([43]_++[43]_-)$
$D=10$	$N=1$	SO_8	$\downarrow SO_8$
		Δ_+	$\downarrow \Delta_-$
		Δ_-	$\downarrow [1]$
	$N=2$	SO_{16}	$\downarrow SO_8\times SO_2$
		Δ_+	$\downarrow ([1^4]_-+[2])[0]+[1^2]([2]_++[2]_-)+[0]([4]_++[4]_-)$
		Δ_-	$\downarrow [\Delta;\,1]_-([1]_++[1]_-)+\Delta_+([3]_++[3]_-)$

Table 7. (continued)

$N=3$	SO_{24}	$\downarrow SO_8\times SO_3$
	Δ_+	$\downarrow \Delta_-[4]+[\Delta;1]_+[3]+[\Delta;1^2]_-[2]+[\Delta;2]_-[1]+[\Delta;1^4]_-[0]$
	Δ_-	$\downarrow [1][4]+[1^3][3]+[21][2]+[21^3]_-[1]+[3][0]$
$N=4$	SO_{32}	$\downarrow SO_8\times SO_4$
	Δ_+	$\downarrow ([2^4]_-+[4])[0]+[31^3]_-([1^2]_++[1^2]_-)+([2^21^2]_-+[31])[2]+[2^2]$ $([2^2]_++[2^2]_-)+[21^2]([31]_++[31]_-)+([1^4]_-+[2])[4]$ $+[1^4]_+([3^2]_++[3^2]_-)+[1^2]([42]_++[42]_-)+[0]([4^2]_++[4^2]_-)$
	Δ_-	$\downarrow [\Delta;21^3]_-[1]+[\Delta;3]_-[1]+[\Delta;21]_-([21]_++[21]_-)+([\Delta;1^3]_-+[\Delta;2]_+)$ $[3]+[\Delta;1^2]_+([32]_++[32]_-)+[\Delta;1]_-([41]_++[41]_-)$ $+\Delta_+([43]_++[43]_-)$
$N=5$	SO_{40}	$\downarrow SO_8\times SO_5$
	Δ_+	$\downarrow \Delta_+[4^2]+[\Delta;1]_+[43]+[\Delta;1^2]_-[42]+[\Delta;1^3]_+[3^2]+[\Delta;2]_-[41]$ $+[\Delta;21]_+[32]+[\Delta;1^4]_-[4]+[\Delta;21^2]_-[31]+[\Delta;2^2]_-[2^2]+[\Delta;3]_+[3]$ $+[\Delta;31]_-[21]+[\Delta;2^21^2]_-[2]+[\Delta;31^3]_-[1^2]+[\Delta;4]_-[1]+[\Delta;2^4]_-[0]$
	Δ_-	$\downarrow [1][4^2]+[1^3][43]+[21][42]+[21^3]_-[3^2]+[21^3]_-[41]+[2^21][32]$ $+[3][4]+[31^2][31]+[32][2^2]+[2^31]_-[3]+[321^2]_-[21]+[41][2]$ $+[41^3]_-[1^2]+[32^3]_-[1]+[5][0]$

Table 8. Decomposition of $\Delta_\pm$ under $SO_{2k}\downarrow SO_{D\times 4}\times U_1\times K$.

$D=1,3\ (\mathrm{mod}\ 8)$	$2k=2^{(D-3)/2}N$
$SO_{2k}\downarrow SO_{D-4}\times U_1\times SO_N$	$D\geqslant 9$
$\Delta_\pm \quad \downarrow\sum_{s_\pm}\sum_{\rho\vdash s_\pm}[\Delta\otimes\{\rho\}]\times\{s_\pm/2-k/4\}\times[\tilde\rho/D]$	
$D=5,7\ (\mathrm{mod}\ 8)$	$2k=2^{(D-3)/2}N$
$SO_{2k}\downarrow SO_{D-4}\times U_1\times Sp_N$	$D\geqslant 7$
$\Delta_\pm \quad \downarrow\sum_{s_\pm}\sum_{\rho\vdash s_\pm}[\Delta\otimes\{\rho\}]\times\{s_\pm/2-k/4\}\times\langle\tilde\rho/B\rangle$	
$D=2\ (\mathrm{mod}\ 8)$	$2k=2^{(D-4)/2}(N_++N_-)$
$SO_{2k}\downarrow SO_{D-4}\times U_1\times SO_{N_+}\times SO_{N_-}$	$D\geqslant 10$
$\Delta_\pm \quad \downarrow\sum_{s_\pm}\sum_{\rho\vdash s_\pm}\sum_{\eta\vdash k-s_\pm}[(\Delta_+\otimes\{\rho\})\cdot(\Delta_-\otimes\{\eta\})]\times\{s_\pm/2-k/4\}\times[\tilde\rho/D]\times[\tilde\eta/D]$	
$SO_{2k}\downarrow SO_{D-4}\times U_1\times SO_N$	
$\Delta_\pm \quad \downarrow\sum_{s_\pm}\sum_{\rho\vdash s_\pm}[\Delta_+\otimes\{\rho\}]\times\{s_\pm/2-k/4\}\times[\tilde\rho/D]$	
$D=6\ (\mathrm{mod}\ 8)$	$2k=2^{(D-4)/2}(N_++N_-)$
$SO_{2k}\downarrow SO_{D-4}\times U_1\times Sp_{N_+}\times Sp_{N_-}$	$D\geqslant 14$
$\Delta_\pm \quad \downarrow\sum_{s_\pm}\sum_{\rho\vdash s_\pm}\sum_{\eta\vdash k-s_\pm}[\Delta_+\otimes\{\rho\})\cdot(\Delta_-\otimes\{\eta\})]\times\{s_\pm/2-k/4\}\times\langle\tilde\rho/B\rangle\times\langle\tilde\eta/B\rangle$	
$SO_{2k}\downarrow SO_{D-4}\times U_1\times Sp_N$	
$\Delta_\pm \quad \downarrow\sum_{s_\pm}\sum_{\rho\vdash s_\pm}[\Delta_+\otimes\{\rho\}]\times\{s_\pm/2-k/4\}\times\langle\tilde\rho/B\rangle$	

The results in table 6 exhaust the possibilities for exploiting isomorphisms and automorphisms. For $D=0,4 \pmod 8$ (16) gives the general result. For other values of D it is possible to make use of the group–subgroup structure

$$\begin{tikzcd}[column sep=small] & \mathrm{SO}_{D-2}\times \mathrm{K} \arrow[dr] & \\ \mathrm{SO}_{2k} \arrow[ur] \arrow[dr] & & \mathrm{SO}_{D-4}\times \mathrm{U}_1\times K. \\ & \mathrm{SU}_k\times \mathrm{U}_1 \arrow[ur] & \end{tikzcd} \qquad (20)$$

The lower chain can be evaluated using first (5) and the properties of plethysm (Littlewood 1950, Wybourne 1970) to yield the results given in table 8. The principal difficulty in implementing those results is the evaluation of the relevant spin plethysms. Having obtained the results for the lower chain the irreps of $\mathrm{SO}_{D-2}\times \mathrm{K}$ that cover those of $\mathrm{SO}_{D-4}\times \mathrm{U}_1\times \mathrm{K}$ can be found by the string method (Wybourne 1984).

Specific results for $D=11$ with $N\leqslant 3$ are given in table 9. For $N=2$ the SO_2 non-scalar irreps are grouped in pairs as for O_2.

Table 9. Branching rules for the basic spin irreps under $\mathrm{SO}_{16N}\downarrow \mathrm{SO}_N\times \mathrm{SO}_9$, $N=1,2,3$.

$D=11$	$N=1$	SO_{16}	$\downarrow \mathrm{SO}_9$
		Δ_+	$\downarrow [2]+[1^3]$
		Δ_-	$\downarrow [\Delta;1]$
	$N=2$	SO_{32}	$\downarrow \mathrm{O}_2\times \mathrm{SO}_9$
		Δ_+	$\downarrow [0]([4]+[31^3]+[31^2]+[3]+[2^3]+[2^21]+[2^2]+[21^3]+[21^2]+[2]$
			$+[1^4]+[1^3]+[1]+[0])$
			$+[2]([31^2]+[31]+[2^21^2]+[21^2]+[21]+[1^3]+[1^2])$
			$+[4]([2^2]+[21^3]+[21]+[2]+[1^4])$
			$+[6]([1^3]+[1^2])$
			$+[8][0]$
		Δ_-	$\downarrow [1]([\Delta;3]+[\Delta;21^2]+[\Delta;21]+[\Delta;2]+[\Delta;1^3]+[\Delta;1^2]+[\Delta;1]+\Delta)$
			$+[3]([\Delta;21]+[\Delta;2]+[\Delta;1^4]+[\Delta;1^2]+[\Delta;1])$
			$+[5]([\Delta;1^2]+[\Delta;1])$
			$+[7]\Delta$
	$N=3$	SO_{48}	$\downarrow \mathrm{O}_3\times \mathrm{SO}_9$
		Δ_+	$\downarrow [0]([6]+[51^2]+[42^2]+[42]+[41^3]+[4]+[3^3]+[3^21]+[32^21]+[321^2]$
			$+2[31^2]+[31]+2[2^3]+[2^21^2]+[2^2]+[21^3]+[2]+2[1^3]+[0])$
			$+[1]([51^3]+[5]+[42^21]+[421^2]+[421]+[41^3]+2[41^2]+[41]+[3^22]$
			$+[3^21^2]+[32^21]+2[32^2]+2[321^2]+2[321]+[32]+3[31^3]+[31^2]+[31]$
			$+[3]+2[2^31]+2[2^21^2]+2[2^21]+2[21^3]+3[21^2]+2[21]+2[1^4]+[1^2]$
			$+[1])$
			$+[2]([51^2]+[51]+[42^2]+[421^2]+[421]+[42]+2[41^3]+[41^2]+[41]$
			$+[4]+[3^221]+[3^21]+2[32^21]+[32^2]$
			$+3[321^2]+3[321]+[32]+2[31^3]+4[31^2]+2[31]+[2^4]+[2^31]+2[2^3]$
			$+3[2^21^2]+2[2^21]+2[2^2]+4[21^3]+2[21^2]+2[21]+2[2]+[1^4]+2[1^3]$
			$+[1^2])$
			$+[3]([421^2]+[421]+[41^3]+2[41^2]+[41]+[3^21^2]+[3^2]+[32^3]$
			$+[32^21]+[32^2]+3[321^2]+2[321]+2[32]+4[31^3]+2[31^2]+2[31]$
			$+2[3]+2[2^31]+[2^3]+3[2^21^2]+3[2^21]+[2^2]+3[21^3]+4[21^2]+2[21]$
			$+2[1^4]+[1^3]+[1^2]+[1])$
			$+[4]([42]+[41^3]+[41]+[4]+[3^21]+[32^21]+2[321^2]+2[321]+[32]$
			$+2[31^3]+3[31^2]+2[31]+[2^4]+[2^31]+[2^3]+3[2^21^2]+2[2^21]+2[2^2]$
			$+4[21^3]+2[21^2]+2[21]+2[2]+[1^4]+2[1^3]+[1^2])$

Table 9. (continued)

	$+[5]([321^2]+[321]+[32]+2[31^3]+[31^2]+[31]+[3]+[2^31]+2[2^21^2]$
	$+2[2^21]+[2^2]+2[21^3]+3[21^2]+2[21]+2[1^4]+[1^3]+[1^2]+[1])$
	$+[6]([31^2]+[31]+[2^3]+[2^21^2]+[2^21]+[2^2]+2[21^3]+[21^2]+[21]$
	$+[2]+[1^4]+2[1^3]+[1^2]+[0])$
	$+[7]([21^3]+[21^2]+[21]+[1^4]+[1^2]+[1])$
	$+[8]([2]+[1^3])$
Δ_-	$\downarrow [0]([\Delta;41^3]+[\Delta;4]+[\Delta;321]+[\Delta;31^2]+[\Delta;31]+[\Delta;2^21]+[\Delta;21^3]$
	$+[\Delta;21^2]+[\Delta;21]+[\Delta;2]+[\Delta;1^4]+[\Delta;1^2]+[\Delta;1])$
	$+[1]([\Delta;5]+[\Delta;41^2]+[\Delta;41]+[\Delta;4]+[\Delta;32^2]+[\Delta;321]+[\Delta;32]$
	$+[\Delta;31^3]+2[\Delta;31^2]+2[\Delta;31]+2[\Delta;3]+[\Delta;2^3]+[\Delta;2^21^2]$
	$+2[\Delta;2^21]+[\Delta;2^2]+[\Delta;21^3]+3[\Delta;21^2]+3[\Delta;21]+2[\Delta;2]+[\Delta;1^4]$
	$+2[\Delta;1^3]+2[\Delta;1^2]+2[\Delta;1]+\Delta)$
	$+[2]([\Delta;41^2]+[\Delta;41]+[\Delta;4]+[\Delta;321^2]+[\Delta;321]+[\Delta;32]$
	$+[\Delta;31^3]+3[\Delta;31^2]+3[\Delta;31]+2[\Delta;3]+[\Delta;2^3]+[\Delta;2^21^2]$
	$+2[\Delta;2^21]+2[\Delta;2^2]+2[\Delta;21^3]+4[\Delta;21^2]+4[\Delta;21]+3[\Delta;2]$
	$+[\Delta;1^4]+3[\Delta;1^3]+3[\Delta;1^2]+2[\Delta;1]+\Delta)$
	$+[3]([\Delta;41]+[\Delta;4]+[\Delta;321]+[\Delta;32]+[\Delta;31^3]+2[\Delta;31^2]$
	$+3[\Delta;31]+2[\Delta;3]+[\Delta;2^31]+[\Delta;2^21^2]+2[\Delta;2^21]+2[\Delta;2^2]$
	$+2[\Delta;21^3]+4[\Delta;21^2]+5[\Delta;21]+3[\Delta;2]+2[\Delta;1^4]+2[\Delta;1^3]$
	$+3[\Delta;1^2]+3[\Delta;1])$
	$+[4]([\Delta;32]+[\Delta;31^3]+[\Delta;31^2]+2[\Delta;31]+2[\Delta;3]+[\Delta;2^21^2]$
	$+[\Delta;2^21]+2[\Delta;2^2]+2[\Delta;21^3]+3[\Delta;21^2]+4[\Delta;21]$
	$+3[\Delta;2]+[\Delta;1^4]+3[\Delta;1^3]+3[\Delta;1^2]+2[\Delta;1]+\Delta)$
	$+[5]([\Delta;31]+[\Delta;3]+[\Delta;2^21]+[\Delta;2^2]+[\Delta;21^3]+2[\Delta;21^2]$
	$+3[\Delta;21]+2[\Delta;2]+[\Delta;1^4]+2[\Delta;1^3]+3[\Delta;1^2]+2[\Delta;1]+\Delta)$
	$+[6]([\Delta;21^2]+2[\Delta;21]+[\Delta;2]+[\Delta;1^4]+[\Delta;1^3]+2[\Delta;1^2]+2[\Delta;1]$
	$+\Delta)$
	$+[7]([\Delta;2]+[\Delta;1^3]+[\Delta;1^2]+[\Delta;1]+\Delta)$
	$+[8][\Delta;1]$

5. Conclusions

The branching rules for several important subgroups of SO_{2k} have been obtained in a compact form using properties of Schur functions. The relevant rules that arise in the determination of light-like representations from extended Poincaré supersymmetry have been given in a form that readily allows extensions to higher helicity states if required.

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